SAHA-S EQUATION OF STATE. Call of interpolation procedure and test example

The aim of the project is to give an example of interpolation of thermodynamic values in the frames of SAHA-S tables.

Input parameters are pressure P (dyne/cm^2) and temperature T (K). Further input parameters are hydrogen mass fraction X, and mass fraction Z of all elements heavier than helium. Presently, B-spline interpolation is performed with respect to variables P, T, and X, and then linear interpolation is performed with respect to Z. Historically, for the sake of time-saving, the logical parameter **`deriv`** was used to control calculations of some derivatives (see below).

Output parameters are density ρ (g/cm^3) together with a wide set of thermodynamic quantities: internal energy U (erg/g), specific heat at constant pressure c_P, adiabatic gradient ∇_{ad}, adiabatic exponent Γ_1, and also the values δ, α, β:

$$\delta = \chi_T / \chi_\rho,$$

$$\alpha = 1 / \chi_\rho,$$

$$\beta = 1 - (a/3)T^4 / P,$$

where $\chi_T = \partial \ln P / \partial \ln T|_\rho$ and $\chi_\rho = \partial \ln P / \partial \ln \rho|_T$. $a \approx 7.5657 \cdot 10^{-15}$ erg cm^{-3} K^{-4} is radiation constant. The derivatives of these functions (except α, β, Γ_1) with respect to T at constant P, as well as to P at constant T are also computed, together with the derivatives with respect to the hydrogen abundance X. Derivatives of δ, c_P, and ∇_{ad} are computed only if input parameter **`deriv=.true.`**

Call of the interpolation procedure is performed as follows:

```
CALL etat_saha(P,T,X,Z,deriv,ro,drop,drot,drox,u,dup,
dut,dux,delta,deltap,deltat,deltax,cp,dcpp,dcpt,dcpx,
gradad,dgradadp,dgradadt,dgradadx,alfa,beta,gamma1)
```

Test example for one point in the solar interior is represented in program

`test3_rho_ptxz.f90`

The test program has to generate file **test_results.dat** with all input and output parameters, like **test_results0.dat** which we give for control :

```
P,T,H,Z
ro,drop,drot,drox,
u,dup,dut,dux,
delta,deltap,deltat,deltax,
cp,dcpp,dcpt,dcpx,
gradad,dgradadp,dgradadt,dgradadx,
alfa,beta,gamma1
  0.23556131246E+18  0.15617057000E+08  0.34672063589E+00  0.19337932155E-01
  0.15300001439E+03  0.64037615513E-15 -0.95431813953E-05 -0.16081486225E+03
  0.23086157328E+16  0.77384105299E-04  0.14670248848E+09  0.24271561571E+16
  0.97409404564E+00 -0.16860010628E-18  0.55792591314E-08  0.80543617427E-02
  0.24273412317E+09 -0.25905595107E-10  0.92473361973E+00  0.25525735329E+09
  0.39562515166E+00 -0.26289756332E-20  0.10252313018E-09  0.30727815704E-02
  0.98593354454E+00  0.99936315630E+00  0.16651196493E+01
```

Files of the FORTRAN project:

1. **test3_rho_ptxz.f90** - main program
2. **mod_etat_saha.f90** - module

Data files:

eos_saha2_000.dat
eos_saha2_010.dat
eos_saha2_015.dat
eos_saha2_020.dat
eos_saha3_Bspl_000.dat
eos_saha3_Bspl_010.dat
eos_saha3_Bspl_015.dat
eos_saha3_Bspl_020.dat

Notes

Our interpolation example uses data files for mass fraction Z=0.015 (**eos_saha2_015.dat, eos_saha3_Bspl_015.dat**) and Z=0.020 (**eos_saha2_020.dat, eos_saha3_Bspl_020.dat**). They are specified in reading soubroutine **z_read2_saha.**

If you want to use other values of Z limits for interpolation, you have to change in the file **mod_etat_saha.f90,** subroutine **z_read2_saha.f90:**

(1) values **Ztab1** and **Ztab2** (possible values are 0.000, 0.010, 0.015, 0.020) – see Lines 212 and 213,
(2) change the names of files according to **Ztab1** (Lines 219 and 246) and **Ztab2** (Lines 231 and258).