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Solar Models with a Helioseismic Sound Speed Profile

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Abstract

The problem of constructing a model for the Sun for a given sound speed in the radiative zone is considered. It is shown that the chemical composition is a free parameter, while the entropy of the adiabatic part of the convection zone is determined in the model construction process. Comparison of parameters for a family of models having classical chemical composition profiles with helioseismic calibrations of the solar envelope reveals a disagreement. Study of the seismic properties of the solar core for the set of models in question indicates that the best agreement with observational data is provided by the model closest to the standard solar model.

1. INTRODUCTION

This paper is concerned with the construction of a model for the internal structure of the Sun, allowing for constraints imposed by helioseismic data. An obvious final aim is the construction of a solar model for which the theoretical oscillation spectrum coincides with the observed spectrum within the observational errors. In spite of considerable success in the interpretation of the spectrum of the five-minute solar oscillations, the study of the internal structure of the Sun is still based largely on the standard solar model (SSM), calculated in the framework of classical assumptions [1, 2].

The construction of a SSM is an entirely closed problem, although it depends on knowledge of at least five functions describing the physics of solar plasma. For this reason, if current conceptions about the physics of the solar interior do not yield agreement between the theoretical and observed oscillation spectra [3,4], we must go beyond the SSM in order to improve the agreement between the spectra, proposing that some of its assumptions are invalid or need corrections.

We consider in this paper a class of solar models extended primarily by rejection of the usually specified opacities in the radiative interior of the star, retaining other classical assumptions (concerning the chemical composition profile, for example) without change, or with minimal changes. In the context of this class of models, we investigate the possibility of construction of a model with specified parameters in the convective envelope and radiative zone, supposing these parameters to be reliably determined from oscillation analysis.

2. THE SEISMIC INFORMATION USED

The observational basis for seismic investigations of the Sun is provided by the frequency spectrum of oscillations, with periods from 3 to 14 min in a wide range of mode degrees from 0 to 200, and even 600 (see [5] for an example of the most representative and widely used data on the frequency spectrum). However, each frequency is an integral characteristic of the structure of the Sun, and the major successes in helioseismic interpretation of observed spectra are due to the possibility of obtaining information about the structure of local regions in the solar interior. An obviously important seismic characteristic of the Sun is the sound speed profile, the determination of which was started in [6] and has recently lead to a most important and well established helioseismic result ([7-10] are among the latest papers on this subject). Because of limitations in the range of available observational data and the finite accuracy of frequency spectrum measurements, the determination of the sound speed profile cannot be made with good accuracy for all regions of the Sun. In this situation, an important source of additional information on the structure of individual regions (for instance, the helium ionization zone) are model calibration methods with an asymptotic approximation [11- 13], the results of which are now in good or close to good agreement (in particular, regarding the helium abundance of the solar convection zone).

Our approach uses secondary helioseismic information in the form of relatively well-defined parameters and functions that characterize the model directly, and attempts to combine these parameters with standard or nearly standard assumptions about solar structure. In this paper, we use three basic results obtained from the analysis of oscillation frequencies:

(1) Determination of parameters for the structure of the helium second ionization zone in the convection zone. These are the helium abundance and specific entropy, considered here as global characteristics of the entire convection zone. We use the results of [12, 14], which are close to the helium abundances of [11, 13].

(2) Determination of the geometrical depth of the convection zone. The theoretical location of the lower boundary of the convection zone may be estimated as the position of the discontinuity in the second derivative of

the sound speed profile (the derivative of the temperature gradient [15]). However, more accurate estimates of the location of this discontinuity are obtained from model calibration, i.e., from the best agreement of specific selected model and observed frequencies. We used the results of [16]; note that the results of [8] agree with these.

(3) The sound speed profile in the radiative zone of the Sun, determined from the oscillation frequencies (the results of [10] are used). It is shown in Section 5 that the results of [8] are in good agreement with this.

The sound speed in the model (given a known adiabatic compressibility modulus) is sufficient for construction of a so-called mechanical model, that is, the density and pressure profiles (see [8] and the discussion in Section 5). We consider in this paper, however, a more general formulation of the problem, when a full solar model is constructed, taking the equations for temperature and luminosity into account, but with a given sound speed. This formulation makes it possible, first, to include constraints associated with the thermal structure of the star (such as the conditions for luminosity, the structure of the convective envelope, and some others), and, second, in the case of success, to immediately obtain a fully (not only mechanical, but also thermal) seismically consistent model.

3. A SOLAR MODEL WITH GIVEN PARAMETERS FOR THE CONVECTIVE ENVELOPE

We will first specify which solar models will be considered. All the models calculated represent full models of a star with solar radius ($R = 6.9599 \cdot 10^{10} cm$), luminosity ($L = 3.846 \cdot 10^{33} erg/s$), and mass ($M = 1.989 \cdot 10^{33} g$), inside which the classical relations of equilibrium (hydrostatic equilibrium and energy conservation) are valid; the basic variables (pressure P, density ρ, temperature T, mass M_r radius r and luminosity L) are assumed to be continuous. The fulfillment of central boundary conditions and joining to an HSRA photosphere model [17] in the outer layers are also assumed. The models have a classical chemical composition profile that is continuous, constant in helium and hydrogen abundance outside the energy-producing core, and in accordance with the generally accepted helium abundance in the envelope (between 20 and 30%). In all cases, the mass fraction of heavy elements is 0.02. The model calculations are performed using the MUSEC code, described with technical calculation details in [18, 19]. In accordance with the terminology adopted in [18], this type of model will be called static (i.e., non evolutionary).

Note the differences between the models considered here and the standard evolutionary solar model. As fol lows from the general theory of internal stellar structure, obtaining a given stellar radius and luminosity requires at least two free internal parameters. Instead of choosing two parameters, we use two functions in calibration: the chemical composition profile and the opacity, assuming some parametrization of variations. This allows consideration of models with other constraints in addition to the given radius and luminosity. The chemical composition profile of the model (in particular, in the core) is not evolutionary, but rather is obtained from the SSM profile (see the SSM descrip tion in [18, 19]) by adding a constant in all parts of the model. We therefore consider these models to be non evolutionaiy. Further, the opacity function, originally taken from the OPAL tables [20], may be subject to modification in various regions of the radiative zone and core. Modifications of opacity near the base of the convection zone are considered in the form of multiplication of the original opacity K by a temperature function of the form

$$1 + a \exp\left(\frac{(\log T - \log T_0)^2}{w}\right), \tag{1}$$

where a is the correction amplitude, w determines the width of the correction range, and $\log T_0$ is the location of its maximum. We use the following values:

$\log T_0 = 7.2$, $w = 0.05$ in the model core

$\log T_0 = 6.6$, $w = 0.05$ in the radiative zone (2)

$\log T_0 = 6.3$, $w = 0.01 - 0.02$ near the base of the convection zone.

Other physical functions (the convective gradient, the rate of energy release, and thermodynamic variables) are assumed to be fixed, and correspond to those supposed in the standard model [18].

Thus, the main distinguishing feature of our models is a largely arbitrary opacity profile in the region of radiative equilibrium of the star, which is equivalent to arbitrary (though relatively small and smooth) variations in the temperature profile.

In this section, we consider the construction of a solar model with specified parameters for the convective envelope as the only constraint. We take for envelope parameters the helium abundance in the envelope Y and the specific entropy in the adiabitic part of the convection zone S (see the discussion regarding calculation of the specific entropy in [18]). Although the quantities Y and S characterize the convection zone, in terms of their physical meaning, they fully correspond to the calibrated parameters in the classical SSM, namely, the initial hydrogen abundance and the convection theory parameter. Therefore, these will be referred to below as parameters of the model as a whole.

Naturally, the construction of a solar model with parameters close to those determined helioseisrnically, that is, with a low helium abundance ($Y = 0.25$) and relatively low entropy ($S/R_g = 21.0 mol/g$), is of interest. As discussed in [18], not a single standard model with published opacities corresponds to these helioseismic values. We therefore use modifications of the opacity in calculating our static solar models. Our calculations rely upon conclusions formulated in [18], namely:

(1) the relation between the opacity in the radiative zone and the entropy in the convective envelope;

(2) the dependence of the envelope helium abundance (in models with a classical chemical composition profile) on the opacity in the core, and on the quantitative constancy of the specific entropy of the model in core modifications;

(3) the weak influence of the opacity near the base of the convection zone (and consequently of the depth of the convective zone) on parameters of the solar model.

Table 1. Model parameters

Model	Convective envelope			Central values			Neutrino fluxes		Coeff. for opac. correc. [eq.(1)]	
	Y	s/R_r	H_{cz}/R	T_c $10^6 K$	ρ_c, g/cm^3	$c^2 \cdot 10^{15}$ cm^3/s^3	Cl	Ga	core	radiat. zone
MI (83)*	0.2499	20.99	0.3067	14.61	180.99	2.4532	2.98	103.3	-0.578	0.0
M2 (80)	0.2507	21.03	0.3063	15.12	162.13	2.5323	4.51	110.0	-0.250	0.510
Mla (64)	0.2496	21.02	0.2874	14.60	181.02	2.4530	2.95	103.1	-0.580	0.0
M2a (81)	0.2504	21.06	0.2870	15.12	162.04	2.5324	4.50	109.9	-0.250	0.510
CCM- -OPAL(105)	0.2803	20.87	0.2810	15.72	150.84	2.5407	8.51	127.4	0	0
M3(113)	0.2400	21.36	0.2862	14.65	169.69	2.4822	2.96	102.6	-0.507	-0.03
M4(112)	0.2503	21.23	0.2876	14.94	164.07	2.5011	3.94	107.6	-0.385	-0.05
M5 (ill)	0.2596	21.10	0.2863	15.14	162.21	2.5077	4.81	111.9	-0.309	-0.04
M6(114)	0.2803	20.82	0.2877	15.69	152.27	2.5376	8.35	126.9	-0.026	-0.07
Observ.**							2.55 ±0.35	77 ±10		

Notes: * The model's identification number is given in parentheses after its name.

** The observed values of the neutrino flux according to [25].

Note that these statements indicate the direction of suitable opacity modifications. The exact opacity cor rection profile, determined non-uniquely in the frame work of such weak constraints, depends on the selected parametrization (1) and (2).

In construction of a model with seismic values of Y and S, we started from a certain SSM, for which the OPAL-SSM was used (according to the notation adopted in [18]). This model already has entropy close to that desired, and the problem reduces to achievement of the required helium abundance. Following the conclusions of [18], we must reduce the opacity in the core while maintaining that of the radiative zone. The required reduction of opacity in the core was nearly 60%. The parameters of this model (M1) are presented in Table 1. Similarly, a model with a seismic envelope was constructed starting from the model ST76-SSM. This model already has the required envelope helium abundance, but its entropy is too high. We may reduce the entropy by increasing the opacity in the radiative zone, but this increases the helium abundance, and a new reduction of the core opacity is required to compensate for this increase. The resulting model is referred to as M2.

Thus, a model with given convection zone characteristics may be constructed in more than one way (which is natural with such little a priori information). The general features of these models are evident, however. First, they must have radiative zone opacity close to the OPAL value. Figure 1 shows the opacities for the models M1 and M2 calculated with inclusion of the opacity corrections made in the construction of these models, but for a fixed set of points in the (T, p) plane. A second general feature is very low core opacity, which assures a iow helium abundance. A relation between core opacity and helium abundance has repeatedly been noted in the literature (see [18] and references therein).

In essence, even from the example of these two models, we have an indication of contradictions in the helioseismic values for the model parameters, at least for models with classical chemical composition profiles. The low helium abundance and low entropy require a substantial decrease in the core opacity, improbable from the standpoint of possible errors in theoretical opacities. Such a decrease is equivalent to a decrease in the temperature gradient (and the sound speed gradient) in the core, and can be based only on the existence of additional mechanisms of heat transfer apart from radiative transfer (for example, weakly interacting massive particles). On the other hand, the low entropy of the model supports a relatively high level of opacity in the radiative zone, approximately corresponding to the OPAL tables [20].

Note also that the methods we used for the opacity correction are far from unique. It is possible to apply a wide class of "wave-like" corrections in the radiative zone, which would result in conservation of the model parameters. The difference between the models M1 and M2 provides an example of opacity variations of this kind (Fig. 1). As shown below, however, the opacity corrections are closely related to the model sound speed profile,

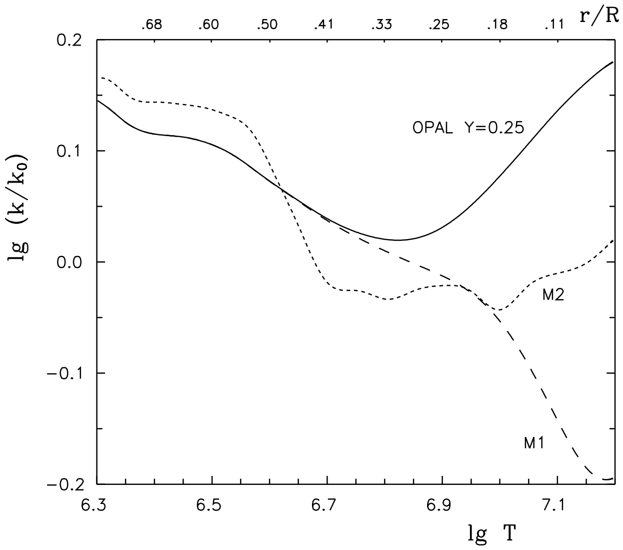

Figure 1: Opacity profiles under conditions typical for the solar model. The logarithm of the opacity ratio and logarithm of a simple analytical law from [21] are both plotted as a function of the logarithm of temperature. The opacities with account of the corrections in M1 and M2, as well as the OPAL profile [20] for $Y = 0.25$, are shown.

which permits reduction of the multitude of possible variations. In turn, the core opacity corrections may be partially equivalent to corrections of the chemical composition profile, as discussed in [18], though the latter cannot fully compensate such substan tial core opacity corrections.

4. THE OPACITY NEAR THE BASE OF THE CONVECTION ZONE AND THE DEPTH OF THIS ZONE

We now turn to the next possibility for refinement of our conceptions about solar structure based on helioseismic results. Let us consider the consequences of measurement of the geometric depth of the convection zone from oscillation frequencies. As already discussed in the Introduction, the issue is consistency of the behavior of the model sound speed gradient and the fre quency spectrum of oscillations. However, if we consider a specified class of models with a standard convection zone base, it is more natural to conduct the analysis in terms of opacity. The basis of our analysis is the relationship between the convection zone parameters Y and S, its depth H_{cz} and the opacity $\kappa(T, \rho, Y)$.

The basic assumption here is that the end of convection occurs in accordance with the Schwarzshild criterion, i.e., for equilibrium between the radiative and adiabatic temperature gradients. We assume that in solar models with a standard description of convection, there exists a unique relation between the above four parameters (or even three of them), valid with accuracy sufficient for our analysis of opacity and seismic properties. To justify this assumption, we rewrite the Schwarzshild criterion in the form

$$\kappa\frac{P}{T^4} = \nabla_{ad}\frac{16\pi acGM_r}{3L}, \tag{3}$$

The right hand side of (3) can be taken to be constant with good accuracy, since the adiabatic gradient near the base of the convective zone is close to 2/5, the luminosity is equal to the total solar luminosity, and the mass differs from the total solar mass by 2% and cannot be substantially changed. For a given chemical composition, however, the function $\kappa P/T^4$ depends only on T and ρ or, alternatively, on temperature and entropy (T, S). Thus, if we know the entropy S and chemical composition, the temperature at the base of the convection zone may be determined with relation (3). If the equation of state is assumed to be close to that of an ideal gas, the dependence of the ratio κ/ρ on S and Y can be expressed in a more explicit form by rewriting the left hand side of (3) in terms of $f(S) = P/T^{2.5}$ and the molecular weight μ:

$$\kappa\frac{P}{T^4} = \frac{\kappa}{\rho}f^2(S)\frac{R_g}{\mu}, \tag{4}$$

The expressions (3) and (4) indicate that there is a relationship between κ/ρ, S, and Y, and this relationship does not include the radius! In other words, knowing S and Y, we can determine κ/ρ at the base of the convection zone.

Another necessary assumption is that the temperature (and density) profile in the adiabatic convection zone is a function only of S and Y, and does not depend on the structure of the outer layers. In order to estimate possible variations, we assume the equation of state to be that for an ideal gas and the envelope mass to be negligible. In this case, the dependence $T(r)$ in the convection zone takes the form

$$T(r) = T_0 + \frac{GM_r\mu\nabla_{ad}}{R_g}\left(\frac{1}{r} - \frac{1}{r_0}\right), \tag{5}$$

This relation makes it evident that the dependence of $T(r)$ on the structure of the outer superadiabatic lay ers and the atmosphere is very weak. For example, changes in convection theory may result in changes in the depth of the outer layers of the order of 50-100 km, which is equivalent to corresponding variation in the initial radius r_0 in (5) (since the temperature and density profiles are specified for a given entropy). However, under conditions characteristic for the solar convection zone, the derivative is

$$\left.\frac{d\ln T}{d\ln r}\right|_{r=0} = \frac{T-T_0}{T}\frac{r}{r-r_0} \simeq 10^{-2},$$

i.e., variations in the initial radius of the order of 10^{-3} give rise to relative variations in temperature at the base of the convection zone of the order of 10^{-5}. In other words, if Y, S, and H_{cz} are considered to be known, the opacity at the base of the convection zone [i.e., at some point (T, ρ)] may be determined. Accordingly, the accuracy of theoretical opacities may be estimated by comparing them with the opacity obtained in this way. In order to represent these estimates visually taking possible uncertainties in Y, S, and H_{cz} into account, we carried out calculations of envelopes for the OPAL opacity tables. The results are shown in Fig. 2, in which the depth of the convection zone is plotted as a function of entropy for various helium abundances; this gives an idea of the convection zone depth for models using the OPAL tables. The parallelogram represents the area of possible errors in helioseismic calibration of the helium abundance and entropy [12]. It is evident that the OPAL tables overestimate the depth of the convection zone for helioseismic values of Y and S (about $H_{cz}/R = 0.305$). This indicates that these opacities are overestimated for temperatures $T = 2.3 \cdot 10^6$ K.

This conclusion is contradictory to that of the previous section, in which it seemed that radiative zone opacities required for the construction of models with low helium abundance and entropy confirmed the correctness of the OPAL opacities. However, analysis of the location of the base of the convection zone leads to the conclusion that the OPAL opacities are overesti mated. This contradiction can be formally resolved. Since local opacity variations at the base of the convection zone do not influence the parameters of the full model, we can construct models (M1a and M2a) with "corrected" convection zone depths without changing Y and S. Thus, the use of an additional constraint the convection zone depth leads to "refinement" of the opacity profile, but this opacity is difficult to justify from a physical point of view.

We note some peculiarities of the above analysis. First, at the base of our reasoning lies the use of an independent determination of the entropy in the convection zone. Until we have the entropy (and naturally the chemical composition) in the convection zone, the agreement of the convective zone depth is purely a fortuitous coincidence. For instance, the standard model SSM-OPAL has a convection zone with a shallower depth (compared to the seismic model) due to a higher helium abundance; this does not imply that we need to increase the opacity.

Second, the analysis does not depend on the structure of the model as a whole and is concerned only with the opacity near the base of the convection zone and assumptions regarding this base.

Third, the analysis is based on the Schwarzshild criterion. The most evident source of disruption of this criterion is "overshooting" at the convection zone base. According to common notions of the nature of over shooting, however, it may lead to relocation of the convection zone base (that is, of the temperature gradient inflection point) deeper in comparison with the location of the point $\Delta_{rad} = \Delta_{ad}$, where Δ_{rad} denotes the radiative temperature gradient, i.e.,

$$\nabla_{rad} \equiv \frac{3LP}{16\pi acT^4 GM_r}\kappa,$$

Since helioseismic determinations give the location of the inflection point precisely, the assumption of consid erable overshooting only worsens the inconsistency: in this case, the point $\nabla_{rad} = \nabla_{ad}$ is located yet closer to the surface, that is, further from the point in the model cal culated using the OPAL opacities.

Fourth, we do not consider the possibility of a large error (exceeding the limits previously indicated) in determination of the convection zone depth from oscil lation frequencies, though the conditions exist for the determined depth to be systematically biased. One example may be the presence of a chemical composition gradient immediately under the base of the convection zone. Such a gradient is encountered in calculations of models allowing for the diffusion of elements in the process of solar evolution. It leads to changes in the sound speed gradient, which may mask or distort the helioseismic estimate. This problem, however, requires special consideration. We now formulate the main point of this section. Independent of our conceptions of the structure of the model as a whole, the analysis of the convection zone depth indicates that we must either decrease the OPAL opacities by 30% for temperatures near $2.3 \cdot 10^6$ K, adopt a large entropy-$S/R_g = 21.3 mol/g$, for

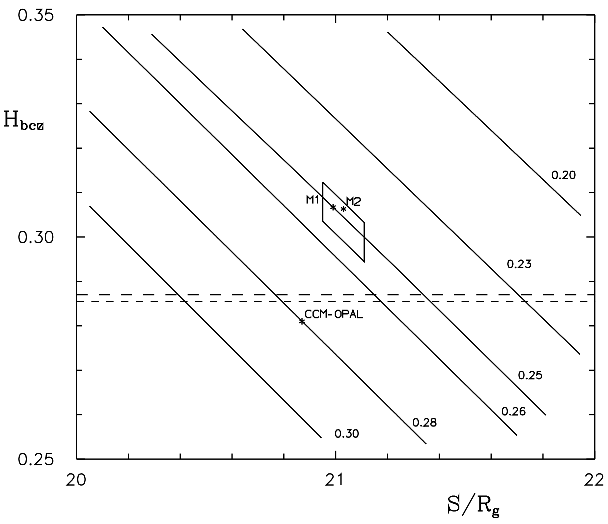

Figure 2: The convection zone depth as a function of entropy and helium abundance in solar envelope models for the OPAL opacity tables. The slanting lines indicate the value of Y. The horizontal dashed lines show the convection zone depth according to [16]. The parallelogram corresponds to the results of helioseismic calibration of the helium ionization zone [12].

example or adopt a higher envelope helium abundance ($Y = 0.26$), depending on which determination of the convection zone depth we assign a higher weight. If we accept the estimates of the entropy and helium abundance in the convection zone, then its seismic depth leads to an inconsistency with the OPAL opacities. If we believe the opacities, we must acknowledge the presence of an error in the evaluation of the helium abundance (by 1%) and/or the entropy (by 0.2 mol/g). It is nevertheless possible to construct a static model with arbitrary opacities.

5. SOLAR MODELS WITH A GIVEN SOUND SPEED PROFILE IN THE RADIATIVE ZONE

Our further aim is to use additional information in the form of the sound speed profile in the radiative zone determined from oscillation frequencies (see [8-10]). In doing so, it is necessary to keep in mind that for a given sound speed profile $c^2(r)$ and known equation of state [known $\Gamma_1(c^2, \rho)$], the mechanical structure of the model (the ρ and P profiles) can be calculated without the radiative transfer law. Indeed, for a known ratio $P/\rho = c^2/\Gamma_1$, the equation of hydrostatic equilibrium may be rewritten

$$\frac{d}{dr}\left\{\frac{r^2}{G\rho}\frac{d}{dr}\left(\frac{\rho c^2}{\Gamma_1}\right)\right\} = -4\pi\rho r^2, \tag{6}$$

In other words, with a precise and complete sound speed profile, we could obtain the model structure accurate to within a non-essential integration constant, for example, in the form of the surface density. [The second integration constant for (6) is fixed by the requirement of regularity of the mass dependence: $M_{(r=0)} = 0$ or $d\rho = 0$ for $r = 0$.] However, even a mechanical model alone cannot be constructed for every profile $c^2(r)$, though a precise formulation of the necessary conditions is quite complicated. We note as an example the condition of a given model mass. If in addition we suppose that the model obtained should have properties characteristic for the Sun, then the relation $d\log(c^2/\Gamma_1)/d\log P = (\Gamma_1 - 1)/\Gamma_1$, should hold in the convection zone, that is, a reasonable equation of state should exist in the form of Γ_1, such that this equation is satisfied for the given sound speed profile; otherwise, it is impossible to construct an adiabatically stratified convection zone. Some (non-rigid) constraints on the sound speed profile near the surface and center [$d(c^2/\Gamma_1)/dr \sim O(r)$ near the center, for example] must also be satisfied. In the context of this paper, however, we suppose that the profile $c(r)$ used satisfies all necessary conditions and permits construction of a solar model. This assumption is used primarily because we will consider this profile locally, only in the radiative zone, ignoring its behavior near both the surface and the center, where it is determined with significant errors, as well as in the convection zone (the last restriction arises primarily due to problems associated with determining the sound speed near the base of the convection zone; see the discussion below).

In reality there are additional constraints on the sound speed profile, associated with the thermal structure of the star, the simplest of which is the total luminosity condition (a set of conditions in a similar formulation of the problem was examined in [22]). The relation between the opacity in the radiative zone and the entropy in the convection zone used here is also a constraint of this type. We therefore consider it necessary to immediately examine full (rather than only mechanical) models, adopting hypotheses about the form of the chemical composition profile. We restrict ourselves here to models with the classical chemical composition profiles specified in

Section 2. As a result, the only unknown function in the radiative zone outside the energy-producing core will be the opacity, which is easy to calculate

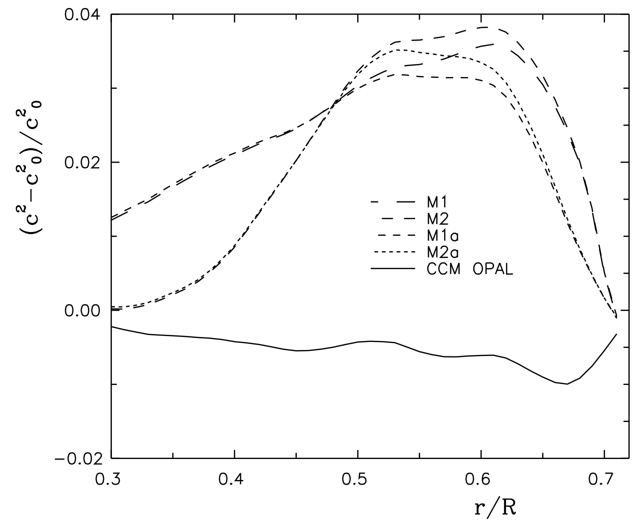

Figure 3: Sound speed profiles in the models M1, M2, M1a. M2a, and SSM-OPAL. c_0^2 is the sound speed profile from [10] determined from solar oscillation frequencies.

using the sound speed gradient [see equation (7) below].

We face a problem, however, associated with the fact that the details of the behavior of the sound speed profile near the base of the convection zone are unknown. This is closely related to the wave character of the oscillations, namely, to the finite and rather large wavelength of the five-minute oscillations in this region. As a result, the resolving capability when reconstructing the profile c(r) from frequencies is modest, and it is impossible to expect an accurate reconstruction of details when asymptotic methods (equivalent to the ray acoustics approximation) are used on a scale substantially less than the wavelength.

In this situation, we use the sound speed profile only in the radiative zone itself, obtaining initial values from a convection zone model that satisfies the seismic constraints to the geometric depth and has a specified helium abundance. In this case, the entropy can no longer be given a priori, since the fixed sound speed profile determines the opacity, and the model can be constructed only for some specific value of the entropy. Such a determination of the entropy for a full solar model will be referred to as model calibration.

We emphasize again the role of a correct convection zone depth in the model. Because only the sound speed gradient is used, the role of the sound speed profile in the convection zone reduces to choosing an initial condition for integration of the sound speed gradient in the radiative zone. In addition, it is well known that the sound speed profiles inside the convection zone are similar in models with different entropies, chemical compositions, etc. (see [23]; a more general formula is given in [24]). Therefore, most important for the evaluation of the initial point is agreement of the geometric depth of the convection zone with the seismic value. We used the value $r_{cz}/R \pm 0.713 + 0.003$ from [16], which agrees within the errors with more recent determinations [4,8].

Note that the relationship used between the opacity and the entropy in full solar models (see Section 3) is equivalent to those between the entropy and the temperature gradient and between the entropy and the sound speed gradient. In this case, the constraints are imposed in a wide range of the radiative zone (to $0.5r/R$). The effect of this constraint on the sound speed significantly exceeds that of divergence in the sound speeds for models differing in their gradient behavior only near the base of the convection zone. The models M1 and M2 provide examples of such geometric disagreement (Fig. 3): the convection zone depth is overestimated by 1.5% compared to M1 and M2 due to opacity corrections, which, however, rapidly fall off. In other words, the opacities (and the gradients) for different models are similar in the deeper layers. (We mention in passing that from the standpoint of the behavior of the temperature gradient, such a local opacity correcion qualitatively imitates the phenomenon of overshooting). It is evident that the effect of the geometric disagreement of the convection zone depths due to local opacity correction is substantially less than the effect of a global opacity correction for the construction of models with a required entropy. These experiments also demonstrate the degree to which the errors in evaluation of the convection zone depth influence our models: the possible errors in determination of the geometric depth (up to 1%) do not effect conclusions concerning the behavior of the sound speed profile in the radiative zone.

Let us proceed to the construction of full models with a sound speed profile. We return to the models M1a and M2a. They have not only given helium abundance and entropy, but also a convection zone depth corresponding to the helioseismic determination. Such an agreement was achieved, how-

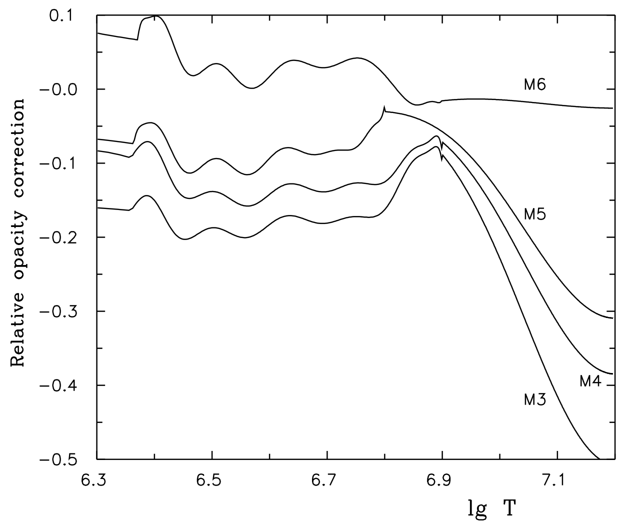

Figure 4: Opacity correction in models M3-M6. The difference with respect to the OPAL tables [20] is shown.

ever, through a complicated opacity correction local decrease near the base of the convection zone while maintaining the OPAL level in the radiative zone. By itself, this opacity correction appears difficult to justify, and leads to the thought that this agreement was obtained in a rather artificial way. The inconsistency in the models M1a and M2a becomes even more evident in comparison of their sound speed profiles to that determined from frequencies: the variation of opacity near the base of the convection zone resulted in a significant enhancement of the sound speed over the helioseismic values (Fig. 3), making these models unacceptable.

For this reason, in order to construct a model with a given sound speed profile and convection zone depth, we must reject specification of envelope parameters, fixing only the helium abundance and determining the entropy from model calibration. In other words, a full solar model with given Y and sound speed profile can be obtained only for a fixed value of entropy in the envelope.

The construction of such a model may be realized in many ways. Technically, the opacity is used as a functional parameter in the model construction process, in order to attain the given sound speed profile.

We used successive approximations for the opacity, enabling us first to find some mean opacity level, in order to obtain a sound speed profile that is on average "correct" and, simultaneously, to construct a full model (in this process, the allowable value of entropy in the envelope is determined). A thin non-systematic opacity correction was then added, which was calculated on the basis of an envelope model with given Y, the value for S obtained, and the formula for the sound speed gradient obtained from the general expression [15] for constant chemical compo sition and the approximation of constant Γ_1:

$$\frac{d\ln c^2}{d\ln P} = 1 - \frac{1}{\Gamma_1} - Q(\nabla_{ad} - \nabla_{rad}), \tag{7}$$

Independent of the method for model construction, we obtain a one-parameter family of models (M3, M4, M5, and M6), differing in their envelope helium abundance, with each giving a specific entropy value and correction for the opacity profile in the radiative zone (Table 1 and Fig. 4). The parameters for the calculated family of models correspond to the dashed line in Fig. 5. It is clear that none in this family of models corresponds to helioseismic calibration of parameters of the helium ionization zone. In addition, these models imply some underestimation of the OPAL table opacities, though this does not exceed 10% in the radiative zone. As already noted, the opacity correction in the core may be the result of heat transfer mechanisms that have not been taken into account.

6. DISCUSSION OF THE GENERALITY OF THE RESULTS

The conclusion that the entropy is fixed in a solar seismic model was obtained on the basis of the results of [18], the most important of which is the fact that variations in opacity and/or chemical composition localized in the core can not alter the entropy of the solar model (and consequently, the opacity and the envelope entropy are interrelated in this model).

Although we cannot generally prove this empirical assertion, it possible to demonstrate that the result con cerning the fixing of entropy considered in this paper is general for a class of solar models, and relies only on the condition of hydrostatic equilibrium. To show this, we begin with the simple argument that for a given chemical composition, the density in the convection zone is a function of entropy. In other words, if we construct a model with a different entropy, it must have a different density profile, with the relative variation of density constant in the adiabatic part of the convection zone to high accuracy (see the model profiles in Fig. 6). We now turn to

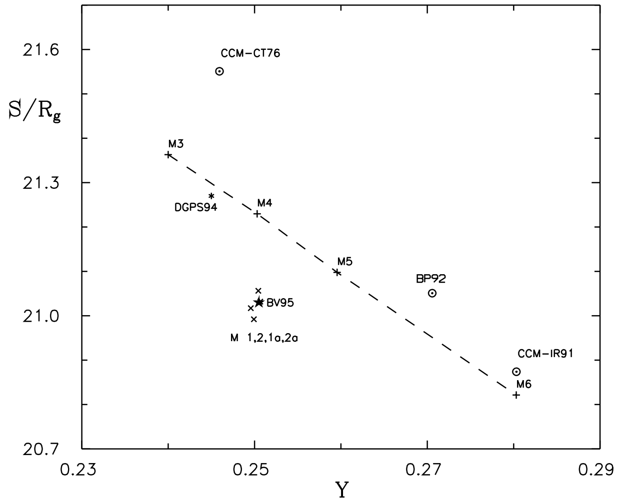

Figure 5: The The location of models in the (Y, S) plane. Shown are models with helioseismic envelope parameters M1 and M2, models with corrected sound speed M3-M6. standard models with the OPAL opacity tables (20) (SSM-OPAL). the ST76 tables [26] (SSM-ST76), the helioseismic determination (asterisk), the seismic model DGPS94 (8].

the formula relating the variations δu $(u \equiv \ P/\rho = c^2/\Gamma_1,)$ and the density variations $\delta\rho$ in two close models satisfying the condition of hydrostatic equilibrium. This formula was obtained in [27] from the linearized equation of hydrostatic equilibrium

$$\frac{d}{dr}\left\{\frac{r^2 u}{G}\frac{d\delta\ln P}{dr}\right\} + \frac{dM_r}{dr}\delta\ln P = \frac{d}{dr}\left\{M_r\frac{\delta u}{u}\right\} + \frac{dM_r}{dr}\frac{\delta u}{u}, \qquad (8)$$

The solution for $\delta\ln P$ has been written in terms of the Green function

$$\delta\ln P = \int\limits_0^R G(r, r^{'})\frac{\delta u}{u}dr, \qquad (9)$$

where the Green function $G(r, r^{'})$ is expressed in terms of the parameters of one of the two models (the explicit expression is given in [27]). Naturally, pressure variations in the adiabatic convection zone are equivalent to

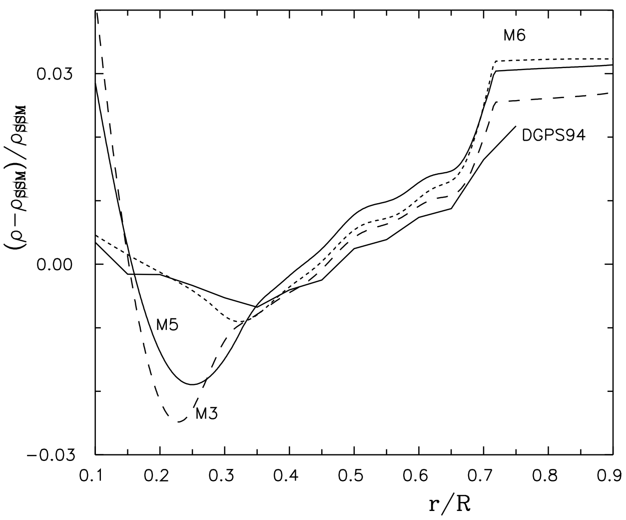

Figure 6: The Density profiles in the models M3, M5, M6, and DGPS94 [8] relative to the density profile in the OPAL-SSM model.

density variations. This means that for models in hydrostatic equilibrium, the entropy in the convection zone (the density profile) is the integral of the sound speed profile over the entire model, with weights determined from the Green function G. However, this conclusion is equivalent to a result derived in consideration of our models; namely, that the entropy in the convection zone is a function of the sound speed in the radiative zone! The relationship between these two quantities is thus valid for all models in hydrostatic equilibrium. (The accuracy of this relationship is limited by the locality of the Green function, i.e., by the weight with which sound speed variations in the core influence a point in the convection zone). A full analysis of the properties of the Green function for solar models falls outside the framework of this paper; we therefore restrict ourselves to this qualitative illustration.

Indeed, comparison of the density profiles in the models (Fig. 6) shows that they are very close in the convective and radiative zones, as could be expected, proceeding from expression (8). This also indicates that the seismic properties of the models are close in the radiative zone, independent of the chemical composition (in particular, the profile of the Brunt-Väisala frequency). Differences are found only in regions where the sound speed is not sufficiently well known: near the center and surface. On the other hand, such a rigid fixation of density in the convection zone implies serious difficulties in matching parameters of the seismic envelope.

In spite of the fact that the above arguments give a basis for generalization of our conclusions to a wide class of models, we should outline the set of models for which these conclusions were derived. The most essential properties are the opacity variations in the models (essentially arbitrary opacity) and the "classical" character of the chemical composition profile. Additional numerical experiments and observation of the properties of already constructed models suggest that the role of the specific functional form of the opacity variations (in particular, in the core) is not great, since the effect of opacity variation in both the core and the convection zone have an integral character. However, the family of models with reduced core opacity considered here by no means exhausts all possible modifications of the core structure. The conditional character of the separation of the opacity variation effect in the core and the radiative zone is more essential. We apply the sound speed profile only down to a certain level in the core, and this level generally coincides with the conditional boundary of energy release. This does not imply, however, that information on the sound speed profile in lower layers is fully unreliable and may not be used; simply in the framework of the model construction procedure used, we cannot (mainly due to the given chemical composition profile) displace this level deeper, since this would deprive us of the possibility of joining our models to solar parameters.

In other words, the class of chemical composition profiles used may be extended. This does not directly affect the results, since variation of the chemical composition in the core and of the opacity do not affect the entropy in the convection zone (in obtaining this result, we also considered models with complicated variations in chemical composition [28]). Variations in the chemical composition in the core, however, may lead to changes in the envelope helium abundance (analogous to the opacity variations); this makes the conclusion of the necessity of opacity reduction in the core for models with seismic sound speed ambiguous.

The most serious difficulties in extending the conclusions obtained to the entire range of modern solar models are associated with the adopted assumptions about the variable character of the chemical composition in the radiative zone. We restrict ourselves here to classic models, in which the hydrogen abundance gradient is zero outside the energy release zone. In reality, there is good reason to believe that the process of gravitational settling of heavy elements during the evolution of the Sun plays a very substantial role, and may result in a hydrogen abundance gradient of 3% between the base of the convection zone and the center [29, 30]. This evidently changes the meaning of the quantities used (in particular, of the entropy and envelope helium abundance) and their connection to the core structure. Therefore, models with gravitational settling and/or diffusion require separate consideration, and are not included in this paper.

7. THE CHARACTERISTICS OF MODEL CORES

Although the major goal this paper is the study of the construction of models with specified envelope parameters and sound speed profile, we will briefly consider the characteristics of the core structure for the models obtained, since they represent a family of models with virtually identical envelopes, but significantly different cores.

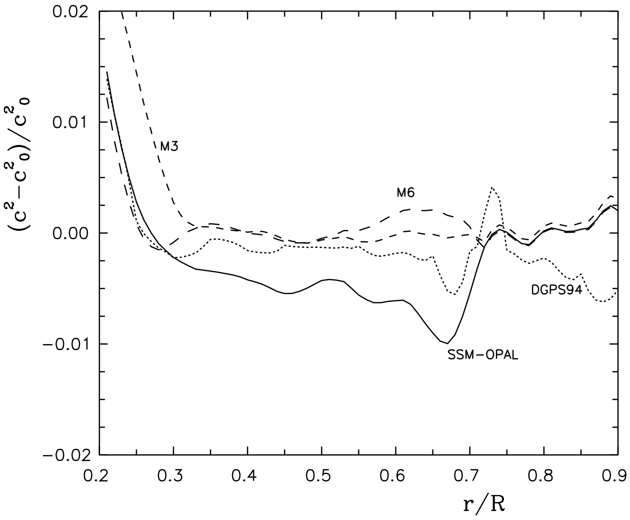

Figure 7: The Sound speed profiles in the models M3, M6, SSM-OPAL, and DGPS94 [8] relative to the sound speed profile from [10] determined from solar oscillation frequencies.

The common tendency in the sequence of models 3-6 is evident: the models have decreasing central temperature and increasing central density (with decreasing helium abundance), which also implies a decrease in the central sound speed (Fig. 7) and increase in the Brunt-Väisala frequency in the outer parts of the core. On comparison of the central sound speed in models M3-M6 (Table 1) with values determined from frequencies ($c^2 = 2.626 \cdot 10^{15} cm^2/s^2$ in [10],$2.673 \cdot 10^{15} cm^2/s^2$ in [8], $2.520 \cdot 10^{15} cm^2/s^2$ in [9]), we conclude that, again, M6 is the best model. However, the scatter in estimates of the sound speed in central regions is still large. The central density of the Sun is also estimated in [8, 9]. Though the reliability of these estimates is not high ($141 - 146.7 g/cm^3$ in [8] and $148 g/cm^3$ in [9]), model M6, which differs only slightly from SSM-OPAL in its global characteristics, proves to be the best with respect to this parameter as well.

We have also calculated the classical characteristics of the neutrino flux and the frequencies of low-degree acoustic oscillations for our models. In calculating the neutrino flux, we used the neutrino absorption cross sections from [31]. and the data from [25] as observed values. The general tendency in the series of models 3-6 is evident: the models have decreasing central tem perature and rapidly decreasing chlorine neutrino flux, in agreement with the observed flux (within the errors) for $T_c = 14.5 \cdot 10^6$ K. The flux of gallium neutrinos, however, is not so sensitive to temperature, and remains enhanced compared to the observed flux.

The relationship between the central value of u and the neutrino flux remains positively correlated (i.e., the larger M, the higher the neutrino flux), in contrast with the paradoxical negative correlation noted for the relatively restricted set of models in [8].

The most simple and commonly accepted seismic test for the core structure is the value of a small splitting for low-degree mode frequencies

$$\delta_l(n) = \nu_{l,n} - \nu_{l+2,n-1}, \tag{10}$$

where l is the mode degree and n is the mode radial order. We calculated oscillation frequencies in the adiabatic approximation taking into account perturbations of the gravitational potential with programs kindly pro vided by S.V. Vorontsov. For the purpose of comparison with observational data, we calculated the values $\delta_0(21)$ and $\delta_1(21)$, and also found linear approximations for the functions $\delta_1(n)$ for n between 15 and 26. The approximation parameters are given in Table 2. Comparison with observational data [32] shows that models with a strongly underestimated temperature and sound speed in the core do not agree with the observed frequencies. In the series of models under consideration, the most satisfactory from the standpoint of seismic properties of the core is M6, closest to the SSM, though it is probable that there is a tendency for a slight decrease in central temperature. Such results are gener ally typical for models with reduced core opacity [33].

Table 2. Small splitting for acoustic oscillation frequencies

Model	$l = 0.2$			$l = 1.3$		
	δ	δ_0	$d\delta/dn$	δ	δ_0	$d\delta/dn$
SSM-OPAL	9.201	9.266	-0.3061	16.150	16.137	-0.4302
M3	8.570	8.657	-0.2950	15.335	15.341	-0.4215
M4	8.463	8.533	-0.2943	15.199	15.212	-0.4295
M5	8.157	8.233	-0.2928	14.799	14.830	-0.4313
M6	9.157	9.210	-0.3034	16.077	16.063	-0.4281
Observ. [32]	8.960	9.040	-0.3044	15.770	15.747	-0.4435

Note: All values are given for n= 21.

8. CONCLUSION

Our major results are the following:

(1) It is shown in the framework of a set of static solar models with classical chemical composition profiles that a model with a helioseismic sound speed pro file must have fully specified values of the convection zone entropy and radiative zone opacity; both quantities are functions of the chemical composition in the envelope.

(2) A discrepancy is revealed in helioseismic and model calibrations of the convection zone entropy (for fixed chemical composition), which indicates either a serious error in the model structure or an error in direct helioseismic estimates of the entropy (and/or chemical composition) in the envelope.

(3) A family of models with seismic sound speed profiles in the radiative zone and core opacity reduced to various extents was constructed. Study of the seismic properties of this family shows that the best model is that with the core opacity reduced by 10% and helium abundance close to that in the standard model. It is also evident that simultaneous agreement with neutrino fluxes and the seismic properties of the core is impossi ble in the framework of this family of models.

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