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ORIGINAL ARTICLE

Smooth models of overshooting at the base of the solar convective zone

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Abstract A set of smoothed temperature gradient profiles around overshooting layers at the solar convective zone bottom is considered. In classical local theories of convection the one point defined according to the Schwarzschild criterion is enough to describe a convective boundary. To get a sophisticated picture of the overshooting we use four points to compute the transition overshooting functions. Analyzing the transition gradient profiles we found that the overshooting convective flux may be either positive or negative. A negative overshooting flux appears in nonlocal convective theories and causes a steep temperature gradient profile. But we propose an evenly smoothed gradient which corresponds to a convective flux positive everywhere. To outline the effect of the temperature gradient on the solar oscillations the squared Brunt–Väisälä frequency N^2 is calculated. In local convective theories the N^2 profile shows the discontinuity of the first derivative at the convective boundary, while all smoothed profiles eliminate the break.

Keywords Sun · Convective zone · Overshooting

1 Definition of the convective temperature gradient

Following the classic mixing-length theory (MLT) of convection (e.g. Cox and Giuli 1968) we define the structure of the convective zone (CZ hereafter) with the logarithmic temperature gradient, $\nabla = \mathrm{d}l\,gT/\mathrm{d}l\,gP$, in terms of temperature, T, and pressure, P. If convective flux is absent, the structure is defined by the radiative gradient, which could be found from the relation $K_\mathrm{r}\nabla_\mathrm{rad} = F_\mathrm{tot} \equiv L_\mathrm{r}/4\pi r^2$; here $K_\mathrm{r} = 4acT^4 g/3\kappa P$ and L_r is luminosity, κ the opacity, a, c, g are the radiation constant, the speed of light and the gravitational acceleration respectively. To compute the gradient ∇ in CZ one needs the adiabatic (i.e. isoentropic) gradient $\nabla_\mathrm{ad} = (\partial lgT/\partial lgP)_s$.

In MLT, the total flux, F_tot, is the sum of the radiative flux, $F_\mathrm{rad} = K_\mathrm{r}\nabla$, and the convective flux, F_c,

$$F_\mathrm{tot} = F_\mathrm{rad} + F_\mathrm{c} \tag{1}$$

and the basic energy equation becomes

$$K_\mathrm{r}(\nabla_\mathrm{rad} - \nabla) = F_\mathrm{c} \tag{2}$$

Equations (1) and (2) are deduced from the MLT assumptions. But we use the expression (2) to calculate the convection flux F_c anywhere, inside or outside of CZ, provided the temperature gradient ∇ is given, instead of calculating of the convective flux F_c from convection theory. For example, MLT gives the form $F_\mathrm{c}^\mathrm{MLT} = K_\mathrm{c}(\nabla - \nabla')^{3/2}$, where ∇' is the parameter of a nonadiabatic deviation in the convective elements and K_c is the coefficient of the convection conductivity; see Cox and Giuli (1968). The expression of $F_\mathrm{c}^\mathrm{MLT}$ is proportional to the averaged velocity of the convective cells, V, and the temperature fluctuation, δT, i.e. $F_\mathrm{c} \simeq c_\mathrm{p}\rho V\delta T$ (here ρ is the density and c_p the specific heat under constant pressure).

2 Points define overshooting region

Following to Shaviv and Salpeter (1973) let us define several points and intervals, applicable in local and nonlocal convection models. The first is the δ_∇-point, where the equation $\nabla = \nabla_\mathrm{ad}$ is accomplished according to the Schwarzschild criterion. This point is the boundary of CZ in MLT. Besides

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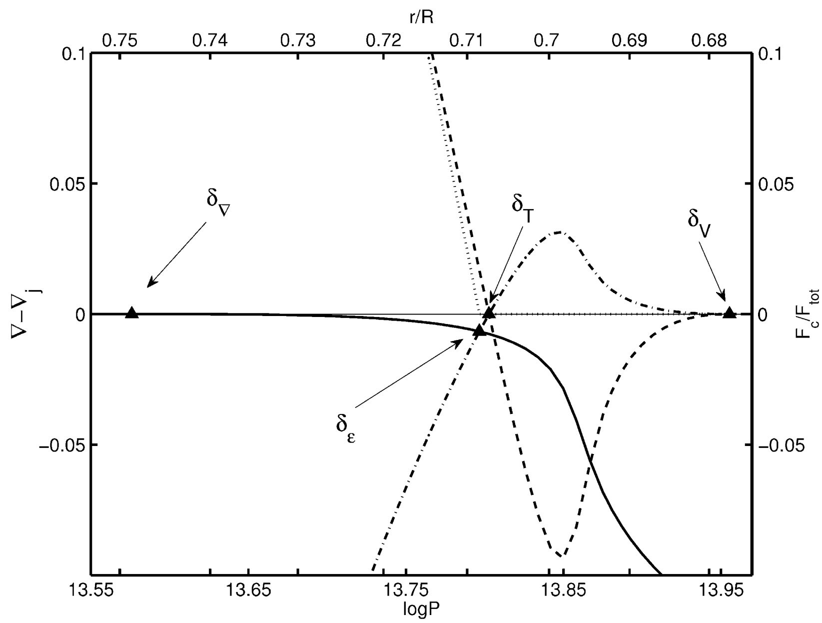

Fig. 1 The *solid line* is difference of the temperature gradient with the adiabatic one, the *dash-dotted line* is the difference with the radiative one, all are plotted versus pressure and radius. *Dashed lines* correspond to fractions of convective flux F_c/F_{tot} in the first case overshooting model (*long dashes*) and in the MLT model (*short dashes*). The definitions of the points are given in the text

of MLT, one point is not enough to describe the transition layers. To generalize the description we introduce another point δ_ϵ, according to the equation $\nabla_{rad} = \nabla_{ad}$; and these two points do not coincide generally. We use δ_ϵ to compare models and define formally the CZ bottom. We introduce two additional points, namely, δ_T and δ_V, where the temperature fluctuation of the convection cells, δT, becomes zero and the convective velocities vanish, $V = 0$, accordingly. All these points coincide in the MLT. However, there are examples of nonlocal convection theories (see e.g. Xiong 1989; Deng and Xiong 2008), describing the overshooting region, where all these four points fall apart.

Let us describe the intervals between the points. In the interval δ_∇–δ_ϵ the gradient obeys $\nabla < \nabla_{ad}$, even within the CZ (the region may be called one of subadiabatic convection). According to (2) the convective flux F_c exceeds the MLT flux F_c^{MLT}. The intervals δ_ϵ–δ_T and δ_T–δ_V are outside of CZ in our classification. Nonlocal convection theory predicts convective flux in these regions, which can be either positive or negative. The sign of convective flux depends on the sign of the correlation between the temperature fluctuations, δT, and the velocities, $\vec{V}$; see Xiong (1989). For the interval δ_ϵ–δ_T the overshooting convective flux is positive, $F_{over} > 0$, while in the region δ_T–δ_V the overshooting flux is negative, $F_{over} < 0$.

3 Examples of the phenomenological overshooting

The MLT gradient $\nabla(r)$ has a derivative peculiarity at the base of CZ, which is partially connected with the coincidence of δ-points. We consider examples of nonlocal theories and construct the corresponding temperature gradient in overshooting layers. The main feature of these models is the smoothness of the gradient in the transition layer, which allows one to avoid the peculiarities of physical quantities, specifically the break of the squared Brunt–Väisälä frequency.

Figure 1 shows the gradients and fluxes in a rather general case, where all four points δ_∇, δ_ϵ, δ_T, δ_V do not coincide, and there are three intervals between them. The first interval is for subadiabatic convection, where $\nabla < \nabla_{ad}$. The second interval (tiny on Fig. 1) lies outside the CZ, between the points δ_ϵ and δ_T. The gradient ∇ is slightly less than the radiative one, $\nabla < \nabla_{rad}$, and some positive convective flux is needed according to (2) (while the MLT flux is zero). The third interval is between the points δ_T and δ_V, where the gradient is greater than the radiative one, $\nabla > \nabla_{rad}$, so it demands a negative convective flux (Fig. 1). This picture is very close to results of the nonlocal theory of convection by Xiong (1989) and Deng and Xiong (2008).

Three possible variants of a smooth profile of the gradient at the bottom of CZ are presented on Fig. 2. The first case is also plotted in Fig. 1 and qualitatively corresponds to Xiong (1989) model. In the second case, the points δ_∇ and δ_ϵ coincide. In the overshooting region below δ_ϵ, the gradient is less than adiabatic, but greater than the radiative one, $\nabla_{rad} < \nabla < \nabla_{ad}$. Points δ_T and δ_V also coincide and mean the end of the overshooting region. In this case the overshooting flux is negative. This model is based on the nonlocal theory by Shaviv and Salpeter (1973), and it was utilized by Skaley and Stix (1991) and Monteiro et al. (2000). The third case corresponds to our model of phenomenological overshooting. The points δ_∇ and δ_ϵ do not coincide. Below δ_ϵ the gradient ∇ is less than both the adiabatic and radiative gradients, $\nabla < \nabla_{rad} < \nabla_{ad}$, through the overshooting region.

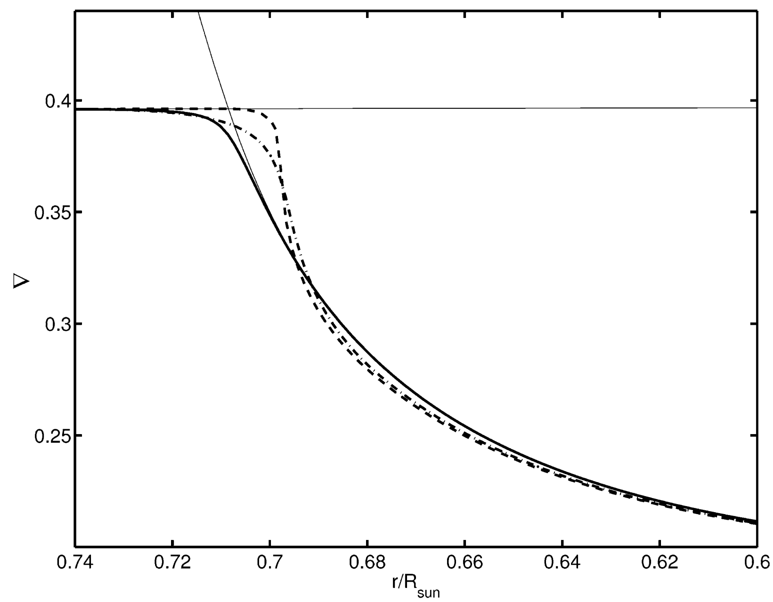

Fig. 2 The smoothed temperature gradients are plotted for the Xiong model (the first case, *dot-dashed line*), the Shaviv and Salpeter model (the second case, *dashed line*) and for our model (the third case, *thick solid line*). Additionally, the adiabatic temperature gradient is plotted by a *horizontal thin line*, and the radiative temperature gradient in our model by a *thin solid line*

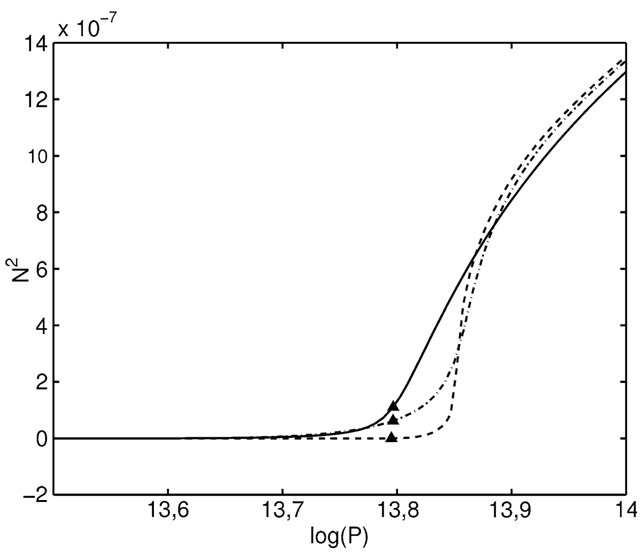

Fig. 3 The squared Brunt–Väisälä frequency is plotted versus pressure for the three cases presented in Fig. 2. *Triangles* are the points δ_ϵ in these models

The convective flux is positive everywhere and exceeds the MLT flux. This model presents an evenly smoothed gradient profile and is consistent with an inertial motion of downward overshooting.

Figure 3 shows the squared Brunt–Väisälä frequency N^2 (the buoyancy frequency named after Väisälä 1925 and Brunt 1927) versus pressure for three cases. The points δ_ϵ (triangles) are plotted to "equalize" the boundary of CZ. By definition N^2 is positive below the point δ_∇. In our model and the model of Deng and Xiong (2008) N^2 becomes positive in the subadiabatic convective region even above the boundary of CZ. In the model of Shaviv and Salpeter (1973), the Brunt–Väisälä frequency changes sign at the point δ_ϵ as in MLT. The steepness of N^2 is connected with the type of overshooting model. To make the N^2-profile less steep than the MLT one, subadiabatic convection and positive overshooting should be assumed.

4 Conclusion

We present a phenomenological description of the bottom of the convective zone and the overshooting region based on smoothed profiles of the temperature gradient. Our models reproduce two important phenomena near the bottom of the convection zone. The first is the subadiabatic structure above the classical boundary of the convection zone, which is explained as inertia of the kinetic downward flux (see Deng and Xiong 2008 for details). The second is an overshooting region.

Models of CZ overshooting are different mainly in the sign of the convective flux. In the model of Shaviv and Salpeter (1973) (case 2), a region of negative convective flux appeared, while our model (case 3) does not assume an inversion flux. The model by Xiong (case 1) (Xiong 1989 and Deng and Xiong 2008) has a combined character and gives a remarkable prediction of the subadiabatic convection within the CZ.

A negative convective flux follows from the possibility of a negative correlation between the speed of the elements, V, and the excess of temperature, δT. If rising ($V > 0$) elements are hotter ($\delta T > 0$) than the environment and lowering elements ($V < 0$) are colder ($\delta T < 0$), then the convective flux is positive. Also, if the temperature gradient ∇ exceeds the adiabatic ∇_{ad}, then the excess of temperature (δT) will increase in absolute value preserving the sign. So the cold elements become even colder (buoyancy correlates with the speed, which is the condition of convective instability). However, below the base of the CZ the temperature gradient is less than the adiabatic one, and the excess of temperature in the element decreases in absolute value. When the path length of elements is sufficiently large, the cold lowering element is compressed and heated and can become hotter than the environment. These "hot" lowering elements transport negative heat flux. In the model of Shaviv and Salpeter (1973) (case 2) the negative flux is a necessary feature, since the only retarding force is buoyancy, and buoyancy is proportional to the excess of temperature. This model should have to be a region of δT inversion, negative flux, and the gradients' behavior as in case 2.

Generally elements are retarded not only by the forces of buoyancy but by turbulent viscosity as well. Then the element may be destroyed before it warms up. In this case convective flux remains positive everywhere and the behavior of the gradient is similar to case 3 (our model). Of course, the exact values of length of path and flux can be obtained only from a detailed hydrodynamic simulation.

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